

Key Stages 2–4

Mathematics

in the National Curriculum for Wales



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Audience	Teachers, headteachers and governing bodies of maintained schools in Wales; local education authorities; initial teacher training providers; teacher unions and school representative bodies; church diocesan authorities; national bodies in Wales with an interest in education.
Overview	This document sets out the Welsh Assembly Government's requirements for mathematics in the national curriculum for Wales. It is issued pursuant to the powers contained in Section 108 of the Education Act 2002 and which are vested in the Welsh Ministers. The Welsh Ministers form part of the Welsh Assembly Government.
Action required	Teachers, headteachers and governing bodies of maintained schools must ensure that the legal requirements set out in this document are implemented in line with the dates specified in the Foreword.
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Additional copies	Can be obtained from: Tel: 029 2037 5427 Fax: 029 2037 5494 Or by visiting the Welsh Assembly Government's website www.wales.gov.uk

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Foreword

This document sets out the revised national curriculum for **mathematics** in Wales.

The structure of the national curriculum

The national curriculum applies to pupils of compulsory school age in maintained schools. It is organised on the basis of three key stages, which are broadly as follows*:

	Pupils' ages	Year groups
Key Stage 2	7–11	3–6
Key Stage 3	11–14	7–9
Key Stage 4	14–16	10–11

In Wales, the following subjects are included in the national curriculum at the key stages shown:

Key Stage 2	English, Welsh, mathematics, science, design and technology, information and communication technology, history, geography, art and design, music and physical education.
Key Stage 3	As at Key Stage 2, plus a modern foreign language.
Key Stage 4	English, Welsh, mathematics, science and physical education.

For each subject, in each of the key stages listed above, programmes of study set out what pupils should be taught and, for Key Stages 2 and 3, attainment targets set out the expected standards of pupils' performance.

At the end of Key Stages 2 and 3, standards of pupils' performance are set out in eight level descriptions of increasing difficulty, with an additional description above Level 8 to help teachers in differentiating Exceptional Performance.

At Key Stage 4, external qualifications are the main means of assessing attainment in the national curriculum. The Welsh Assembly Government publishes annually the list of qualifications that, under Section 96 of the Learning and Skills Act 2000, are approved for use with pupils of compulsory school age.

* The key stages are defined precisely in Section 103 of the Education Act 2002.

Including all learners

The revised national curriculum contains a section on including all learners which clarifies learner entitlement and schools' responsibilities.

Implementation dates

The revised programmes of study and attainment targets for **mathematics** become legal requirements by means of an Order made by the Welsh Assembly Government and come into effect on:

- 1 August 2008 for Years 3, 4 and 5 and Years 7 and 8
- 1 August 2009 for Year 6 and Year 9
- 1 August 2010 for Year 10 in Key Stage 4
- 1 August 2011 for Year 11 in Key Stage 4.

From these dates the existing national curriculum for **mathematics** is superseded.

Welsh Assembly Government
January 2008

Including all learners

Responsibilities on schools

Under the United Nations Convention on the Rights of the Child and the Welsh Assembly Government's overarching strategy document, *Rights to Action*, all children and young people must be provided with an education that develops their personality and talents to the full. The Education Act 2002 further strengthens schools' duty to safeguard and promote the welfare of all children and young people.

The equal opportunities legislation, which covers age, disability, gender, race, religion and belief, and sexual orientation, further places a duty on schools in Wales towards present and prospective learners to eliminate discrimination and harassment, to promote positive attitudes and equal opportunities and to encourage participation in all areas of school life.

Schools should develop in every learner a sense of personal and cultural identity that is receptive and respectful towards others. Schools should plan across the curriculum to develop the knowledge and understanding, skills, values and attitudes that will enable learners to participate in our multi-ethnic society in Wales. Schools should develop approaches that support the ethnic and cultural identities of all learners and reflect a range of perspectives, to engage learners and prepare them for life as global citizens.

Schools must work to reduce environmental and social barriers to inclusion and offer opportunities for all learners to achieve their full potential in preparation for further learning and life. Where appropriate, schools will need to plan and work with specialist services to ensure relevant and accessible learning experiences.

For learners with disabilities in particular, they should:

- improve access to the curriculum
- make physical improvements to increase participation in education
- provide information in appropriate formats.

Schools should seek advice regarding reasonable adjustments, alternative/adapted activities and appropriate equipment and resources, which may be used to support the full participation of all learners, including those who use a means of communication other than speech.

For learners whose first language is neither English nor Welsh, schools should take specific action to help them learn both English and Welsh through the curriculum. Schools should provide learners with material that is appropriate to their ability, previous education and experience, and that extends their language development. Schools should also encourage the use of learners' home languages for learning.

Learner entitlement

Schools in Wales should ensure that all learners are engaged as full members of their school communities, accessing the wider curriculum and all school activities and working wherever possible alongside their peers. Schools should teach all programmes of study and frameworks in ways appropriate to learners' developing maturities and abilities and ensure that learners are able to use fully their preferred means of communication to access the curriculum. In order to extend their learning, learners should experience a variety of learning and teaching styles.

To enable all learners to access relevant skills, knowledge and understanding at an appropriate level, schools may use content from earlier phases or key stages within the curriculum. Schools should use material in ways suitable for the learners' age, experience, understanding and prior achievement to engage them in the learning process.

For learners working significantly below the expected levels at any key stage, schools should use the needs of the learner as a starting point and adapt the programmes of study accordingly. Sufficient flexibility exists within the curriculum to meet the needs of learners without the need for disapplication. In exceptional cases, individual learners may be disapplied, usually on a temporary basis, but group or large-scale disapplications should not be used.

Where it is not possible to cover the content of all of the programmes of study for each key stage, the statutory requirement to provide a broad, balanced curriculum can be met by selecting appropriate topics/themes from the curriculum as contexts for learning.

For more-able and talented learners working at higher levels, schools should provide greater challenge by using material in ways that extend breadth and depth of study and opportunities for independent learning. The level of demand may also be increased through the development and application of thinking, and communication, ICT and number skills across the curriculum.

Schools should choose material that will:

- provide a meaningful, relevant and motivating curriculum for their learners
- meet the specific needs of their learners and further their all-round development.

Learners of all abilities should have access to appropriate assessment and accreditation.

Skills across the curriculum

A non-statutory *Skills framework for 3 to 19-year-olds in Wales* has been developed in order to provide guidance about continuity and progression in developing thinking, communication, ICT and number for learners from 3–19.

At Key Stages 2 and 3, learners should be given opportunities to build on skills they have started to acquire and develop during the Foundation Phase. Learners should continue to acquire, develop, practise, apply and refine these skills through group and individual tasks in a variety of contexts across the curriculum. Progress can be seen in terms of the refinement of these skills and by their application to tasks that move from: concrete to abstract; simple to complex; personal to the 'big picture'; familiar to unfamiliar; and supported to independent and interdependent.

For 14–19 learners, the framework should provide the basis for making effective progress in these skills, which can be assessed through a range of qualifications, including Key Skills.

Developing thinking



Learners develop their thinking across the curriculum through the processes of **planning**, **developing** and **reflecting**.

In **mathematics**, learners ask questions, explore alternative ideas and make links with previous learning in order to develop strategies to solve problems. They gather, select, organise and use information, and identify patterns and relationships. They predict outcomes, make and test hypotheses, reason mathematically when investigating, and analyse and interpret mathematical information. They describe what they have learned, reflect on their work by evaluating their results in line with the original problem, and justify their conclusions and generalisations.

Developing communication



Learners develop their communication skills across the curriculum through the skills of **oracy**, **reading**, **writing** and **wider communication**.

In **mathematics**, learners listen and respond to others. They discuss their work with others using appropriate mathematical language. They read and extract information from mathematical texts. When solving problems, they present their findings and reasoning orally and in writing, using symbols, diagrams, tables and graphs as appropriate.

Developing ICT

Learners develop their ICT skills across the curriculum by **finding, developing, creating and presenting information and ideas** and by using a wide range of equipment and software.

In **mathematics**, learners use a variety of ICT resources to find, select, organise and interpret information, including real-life data, to explore relationships and patterns in mathematics, to make and test hypotheses and predictions, to create and transform shapes, and to present their findings using text, tables and graphs.

Developing number

Learners develop their number skills across the curriculum by **using mathematical information, calculating, and interpreting and presenting findings**.

In **mathematics**, learners use their number skills throughout the programme of study when solving problems in a variety of practical and relevant contexts and when investigating within mathematics itself.

Learning across the curriculum

At Key Stages 2 and 3, learners should be given opportunities to build on the experiences gained during the Foundation Phase, and to promote their knowledge and understanding of Wales, their personal and social development and well-being, and their awareness of the world of work.

At Key Stage 4, learners' knowledge and understanding should be developed and applied within the contexts of their individual 14–19 pathways including the Learning Core.

Curriculum Cymreig (7–14) and Wales, Europe and the World (14–19)



Learners aged 7–14 should be given opportunities to develop and apply knowledge and understanding of the cultural, economic, environmental, historical and linguistic characteristics of Wales. Learners aged 14–19 should have opportunities for active engagement in understanding the political, social, economic and cultural aspects of Wales as part of the world as a whole. For 14–19 learners, this is a part of their Learning Core entitlement and is a requirement at Key Stage 4.

Mathematics contributes to the Curriculum Cymreig by offering learners the opportunity to learn and apply mathematics in the context of data from their own local community, from the local and national environment, and from current issues related to Wales. The traditional Welsh vocabulary for some numbers as well as Welsh quilt and Celtic patterns provide investigative opportunities to contribute to learners' development of a sense of Welsh identity.

Personal and social education



Learners should be given opportunities to promote their health and emotional well-being and moral and spiritual development; to become active citizens and promote sustainable development and global citizenship; and to prepare for lifelong learning. For 14–19 learners this is a part of their Learning Core entitlement and is a requirement at Key Stage 4.

Mathematics contributes to learners' personal and social education by providing opportunities to apply mathematics to real-life problems. It helps them to analyse and interpret information presented to them on environmental and other twenty-first century issues, and to develop an informed and challenging attitude to real-life information, questioning its validity and recognising its implications for their world.

Careers and the world of work



Learners aged 11–19 should be given opportunities to develop their awareness of careers and the world of work and how their studies contribute to their readiness for a working life. For 14–19 learners this is a part of their Learning Core entitlement and is a requirement at Key Stage 4.

Mathematics contributes to learners' awareness of careers and the world of work by providing opportunities to apply mathematics in the context of financial awareness of employment, budgeting, saving and spending.

Progression in mathematics

Mathematical Development in the Foundation Phase

During the Foundation Phase, children develop their skills, knowledge and understanding of mathematics through oral, practical and play activities. They enjoy using and applying mathematics in practical tasks, in real-life problems, and within mathematics itself. They use a variety of ICT resources as tools for exploring number, for obtaining real-life data and for presenting their findings.

Much of their work will be oral. They develop their use and understanding of mathematical language in context, through communicating/talking about their work. They ask and respond to questions, and explore alternative ideas. They use appropriate mathematical language to explain their thinking and the methods they use to support the development of their reasoning. They develop a range of flexible methods for working mentally with number, in order to solve problems from a variety of contexts, checking their answers in different ways, moving on to using more formal methods of working and recording when they are developmentally ready. They explore, estimate and solve real-life problems in both the indoor and outdoor environment. They develop their understanding of measures, investigate the properties of shape and develop early ideas of position and movement through practical experiences. They sort, match, sequence and compare objects and events, explore and create simple patterns and relationships, and present their work in a variety of ways.

Mathematics at Key Stage 2

At Key Stage 2, learners build on the skills, knowledge and understanding they have already acquired during the Foundation Phase. They continue to develop positive attitudes towards mathematics and extend their mathematical thinking by solving mathematical problems, communicating and reasoning mathematically using contexts from across the whole range of mathematics, across the curriculum and as applied to real-life problems.

They extend their use of the number system, moving from counting reliably to calculating fluently with all four number operations, including in the context of money, in order to solve numerical problems. They try to tackle a problem with a mental method before using any other approach and use written methods of calculation appropriate to their level of understanding. They develop estimation strategies and apply these to check calculations, both written and by calculator. They explore a wide variety of shapes and their properties and, in the context of measures, use a range of units and practical equipment with increasing accuracy. They collect, represent and interpret data for a variety of purposes. They select, discuss, explain and present their methods and reasoning using an increasing range of mathematical language, diagrams and charts.

Mathematics at Key Stage 3

At Key Stage 3, learners build on the skills, knowledge and understanding they have already acquired at Key Stage 2, and take on increasing responsibility for planning and executing their work. They maintain positive attitudes towards mathematics. They extend their mathematical thinking by solving mathematical problems, working with increasing confidence and flexibility in solving unfamiliar problems or problems in unfamiliar contexts, communicating their work orally and in a variety of written forms, and reasoning mathematically, explaining their reasoning to others, using contexts from across the whole range of mathematics and as applied to real-life problems.

They extend their calculating and measuring skills, applying them to a wider range of tasks. They begin to use algebraic techniques with confidence, generating and manipulating algebraic expressions and equations. They explore a variety of situations that lead to algebraic or graphical representation. They extend their understanding of reasoned argument when they encounter a variety of examples of simple algebraic and geometric proof. They develop a deeper understanding of the properties of shape, position and movement. They collect, represent, analyse and interpret realistic data, communicating and presenting their findings clearly and concisely using appropriate mathematical language, notation and forms of representation.

Mathematics at Key Stage 4

At Key Stage 4, learners build on the skills, knowledge and understanding they have already acquired at Key Stage 3, and take on increasing responsibility for planning and executing their work. They bring previous experience and related knowledge to bear when considering mathematical or real-life problems, and extend their understanding of the importance of precision and rigour in mathematics. They communicate their findings with increasing clarity and conciseness when using mathematical language and notation, and demonstrate increasing confidence when using spoken language.

They consolidate their knowledge and understanding of the number system and number relationships, and develop their calculation skills, enabling them to solve problems in a variety of contexts, including work-related and vocational areas, and everyday situations, particularly the management of personal finances. They establish greater proficiency in the use of symbolic manipulation, and rigorous mathematical reasoning in algebraic and geometric contexts. Learners engage in purposeful activities in order to extend their understanding of the use of measures and the properties of shape, position, movement and transformation. Through a variety of meaningful contexts, they develop increased confidence in processing and interpreting data, and in understanding and calculating estimates of probabilities.



Skills

Pupils should develop their application and understanding of their mathematical skills using contexts and techniques from across the Range.

1. Solve mathematical problems

Pupils should be given opportunities to:

- select and use the appropriate mathematics, materials, units of measure and resources to solve problems in a variety of contexts
- identify, obtain and process information needed to carry out the work
- develop their own mathematical strategies and ideas and consider those of others
- try different approaches; use a variety of strategies, sequences of operation and methods of calculating
- use their prior knowledge to find mathematical facts that they have not learned, and to solve numerical problems
- use flexible and effective methods of computation and recording
- estimate solutions to calculations; use alternative strategies to check the accuracy of answers
- appreciate the continuous nature of measures, and that measurement is approximate; estimate measures, and measure to an appropriate degree of accuracy in a range of contexts.

2. Communicate mathematically

Pupils should be given opportunities to:

- use correct mathematical language, notation, symbols and conventions to talk about or to represent their work to others
- recognise, and generalise in words, patterns that arise in numerical, spatial or practical situations
- visualise and describe shapes, movements and transformations
- read information from charts, diagrams, graphs and text
- use a variety of methods to represent data
- devise and refine their own ways of recording
- explain strategies, methods, choices and conclusions to others in a variety of ways – verbally, graphically, using informal written methods.

3. Reason mathematically

Pupils should be given opportunities to:

- develop a variety of mental and written strategies of computation
- check results and interpret solutions to calculations, including calculator displays; check against the context of the problem that solutions are reasonable
- develop early ideas of algebra and mathematical structure by exploring number sequences and relationships; explain and predict subsequent terms
- investigate and generalise repeating patterns and relationships; search for pattern in their own results
- present and interpret a wide range of graphs and diagrams that represent data; draw conclusions from this data; recognise that some conclusions can be uncertain or misleading
- make and investigate mathematical hypotheses, predictions and conjectures.



Range

Pupils develop their mathematical skills, knowledge and understanding through learning about and using Number, Measures and money, Shape, position and movement, and Handling data. They should use a variety of ICT resources as tools whenever appropriate.

Number

Pupils should be given opportunities to:

1. Understand number and number notation

- count, read, write and order whole numbers
- understand place value in relation to the position of digits; multiply and divide numbers by 10 and 100
- identify negative numbers and decimals on a number line
- use negative numbers in the context of temperature, and decimals in the context of money and measures

2. Calculate in a variety of ways

- use a variety of mental methods of computation; extend informal written methods to non-calculator methods
- round answers to calculations to an appropriate degree of accuracy
- use the relationships between the four operations, including inverses; recognise situations to which the different operations apply
- use fractions and percentages to estimate, describe and compare proportions of a whole; calculate fractions and percentages of quantities

3. Investigate patterns and relationships

- explore features of numbers, including number bonds, factors, multiples, even and odd numbers, primes, squares and square roots, and sequences of whole numbers
- explore the inverse relationships of addition and subtraction, and of multiplication and division
- deepen their understanding of one-to-one correspondence.

Measures and money

Pupils should be given opportunities to:

1. Understand and use measures

- choose appropriate standard units of length, mass, volume and capacity, temperature, area and time
- understand the relationships between units, and convert one metric unit to another
- read times on analogue and digital clocks; use timetables and convert between the 12- and 24-hour clocks; calculate time differences
- know the rough metric equivalents of imperial units still in daily use
- interpret numbers on scales and read scales to an increasing degree of accuracy; understand and use scale in simple maps and drawings
- draw and measure angles
- find perimeters of simple shapes; find areas and volumes by counting and other practical methods

2. Understand and use money

- know and use the conventional way to record money
- find approximate solutions to, and use the four operations to solve, problems involving money
- understand a calculator display in relation to money, *e.g. that a display of 21.4 (pounds) means £21.40*
- be aware of other currencies.

Shape, position and movement

Pupils should be given opportunities to:

1. Understand and use the properties of shapes

- make 2-D and 3-D shapes and patterns with increasing accuracy
- understand the congruence of 2-D shapes
- name and classify 2-D shapes according to side and angle properties
- know and use the properties of 2-D (polygon) and common 3-D (polyhedron) shapes

2. Understand and use the properties of position and movement

- recognise reflective and rotational symmetries of 2-D shapes
- use positive co-ordinates to specify location
- identify properties of position and movement, and use these to classify shapes
- use right angles, fractions of a turn and degrees to measure rotation.

Handling data

Pupils should be given opportunities to:

1. Collect, represent and interpret data

- collect data for a variety of defined purposes, including those that arise from their own questions, and from a variety of sources
- use and present data in a variety of ways including tables, pictograms, charts, bar charts, line graphs, diagrams, text and ICT
- calculate and use the mode, median, mean and range of a set of discrete data

2. Understand and use probability

- use everyday language for early ideas of probability
- know that the likelihood of an event lies between impossible and certain.



Skills

Pupils should develop their application and understanding of their mathematical skills using contexts and techniques from across the Range.

1. Solve mathematical problems

Pupils should be given opportunities to:

- select, organise and use the mathematics, resources, measuring instruments, units of measure, sequences of operation and methods of computation needed to solve problems
- identify what further information or data may be required in order to pursue a particular line of enquiry; formulate questions and identify sources of information
- develop and use their own mathematical strategies and ideas and consider those of others
- select, trial and evaluate a variety of possible approaches; break complex problems into a series of tasks
- use their knowledge of mathematical relationships and structure to derive facts that they have not yet learned, and to solve numerical problems
- use a range of mental, written and calculator computational strategies
- use a variety of checking strategies, including mental estimation, approximation and inverse operations
- develop their skills of estimating and measuring; recognise limitations on the accuracy of data and measurement; select an appropriate degree of accuracy.

2. Communicate mathematically

Pupils should be given opportunities to:

- use a wide range of mathematical language, notation, symbols and conventions to explain and communicate their work to others
- generalise and explain patterns and relationships in words and symbols; express simple functions in words and symbolically
- visualise, describe and represent shapes, movements and transformations, using related mathematical language
- read mathematical forms of communication, including tables, diagrams, graphs, mathematical texts and ICT
- present work clearly, using diagrams, labelled graphs and symbols
- evaluate different forms of recording and presenting information, taking account of the context and audience
- explain strategies, methods, choices, conclusions and reasoning to others in a variety of ways, including orally, graphically and in writing.

3. Reason mathematically

Pupils should be given opportunities to:

- extend mental methods of computation to consolidate a range of non-calculator methods
- justify how they arrived at a conclusion to a problem; give solutions in the context of the problem; confirm that results are of the right order of magnitude
- interpret and use simple algebraic relationships and functions; predict subsequent terms or patterns in number or geometric sequences
- understand general algebraic statements; make and test generalisations; recognise particular examples of a general statement
- interpret mathematical information presented in a variety of forms; draw inferences from graphs, diagrams and statistics; recognise that some conclusions and graphical representations of data can be misleading; examine critically, improve and justify their choice of mathematical presentation
- explain, follow and compare lines of mathematical argument; make conjectures and hypotheses, design methods to test them, and analyse results to see whether they are valid; appreciate the difference between mathematical explanation and experimental evidence; recognise inconsistencies and bias
- evaluate results by relating them to the initial question or problem; develop an understanding of the reliability of results; recognise that inferences drawn from data analysis may suggest the need for further investigation.



Range

Pupils develop their mathematical skills, knowledge and understanding through learning about and using Number, Measures and money, Algebra, Shape, position and movement, and Handling data. They should use a variety of ICT resources as tools whenever appropriate.

Number

Pupils should be given opportunities to:

1. Understand number and number notation

- extend their knowledge of the number system, including decimals, ratios, fractions, percentages and the relationships between them
- use place value in whole numbers and decimals in computation and metric measurement
- use negative numbers in a variety of contexts

2. Calculate in a variety of ways

- use a calculator efficiently to plan a complex calculation
- calculate with whole numbers, negative numbers, decimals, fractions, percentages and ratios, understanding the effects of the operations

3. Investigate patterns and relationships

- examine features of numbers, including primes, powers and roots
- explore number and geometric patterns and sequences, *e.g. patterns in the natural world, Fibonacci sequences, and the Golden ratio*

- explore inverse number relationships, and use these in computation strategies.

Measures and money

Pupils should be given opportunities to:

1. Understand and use measures

- extend their understanding of the nature of measurement, including the difference between discrete and continuous measures
- make sensible estimates of length, mass, capacity and time in everyday situations, extending to less familiar contexts; calculate time and temperature differences
- develop their understanding of the relationships between units, converting from one metric unit to another
- know imperial measures in common use and their approximate metric equivalents
- use compound measures, including metric or common imperial units where appropriate
- use and interpret scale on graphs, maps and drawings
- read and interpret scales on measuring instruments and understand the degree of accuracy that is possible, or appropriate, for a given purpose
- find perimeters, areas and volumes of common shapes

2. Understand and use money

- understand and use the conventional way of recording money
- calculate with money and solve problems related to budgeting, saving and spending, and currency exchange rates
- interpret a calculator display in relation to money.



Algebra

Pupils should be given opportunities to:

1. Understand and use algebraic relationships, functions and graphs

- extend their knowledge of number operations and relationships to develop the ideas of algebra; appreciate the use of letters to represent variables or unknowns
- experience ways in which algebra can be used to model real-life situations and solve problems
- generate and generalise simple number sequences
- construct and interpret tables and graphs of linear and simple quadratic functions; explore the properties of linear functions and their graphs
- construct, interpret and evaluate formulae and expressions given in words or symbols
- form, manipulate and solve linear equations and inequalities and simple linear simultaneous equations; solve simple quadratic equations, including trial-and-improvement methods where appropriate.

Shape, position and movement

Pupils should be given opportunities to:

1. Understand and use the properties of shapes

- explore properties of shapes through drawing and practical work; construct 2-D and 3-D shapes from given information
- understand congruence of simple 2-D shapes
- explain and use angle properties of 2-D shapes
- explore the properties of polygons and polyhedra; use these to classify shapes
- use Pythagoras' theorem to solve problems

2. Understand and use the properties of position and movement

- use line and rotational symmetries to solve problems in two and three dimensions
- use Cartesian co-ordinates to specify location
- develop their understanding of tessellations and geometric patterns, *e.g. wallpaper patterns, Islamic and Celtic designs, quilting patterns*, and simple transformations of translation, reflection, rotation and enlargement.



Handling data

Pupils should be given opportunities to:

1. Collect, represent, analyse and interpret data

- use a variety of means to collect data in order to follow lines of enquiry or to test hypotheses, *e.g. the internet, questionnaires, data collection sheets, experiment*
- begin to develop an understanding of bias and reliability
- construct appropriate diagrams and graphs to represent discrete and continuous data, including bar charts, line graphs, pie charts, frequency polygons and scatter diagrams
- calculate or estimate values of the mode, median, mean and range of sets of discrete, grouped and continuous data
- interpret information given in a wide range of graphs, diagrams and statistics, especially real-life data
- compare sets of data and their distributions, including methods that involve correlation and lines of best fit

2. Understand and use probability

- understand and use the vocabulary of probability and the probability scale from 0 to 1 through experience, experiment and theory
- recognise situations where probabilities can be based on equally likely outcomes and others where estimates must be based on experimental evidence; use relative frequency over a number of trials as an estimate of probability
- identify all the outcomes of a combination of two experiments or a compound event.

Attainment target

Level descriptions

The following level descriptions describe the types and range of performance that pupils working at a particular level should characteristically demonstrate. In deciding on a pupil's level of attainment at the end of a key stage, teachers should judge which description best fits the pupil's performance. Each description should be considered in conjunction with the descriptions for adjacent levels.

By the end of Key Stage 2, the performance of the great majority of pupils should be within the range of Levels 2 to 5, and by the end of Key Stage 3 within the range 3 to 7. Level 8 is available for very able pupils and, to help teachers differentiate Exceptional Performance at Key Stage 3, a description above Level 8 is provided.

Level 1

Pupils use mathematics as an integral part of classroom activities. They represent their work with objects or pictures and discuss it. They count, order, add and subtract numbers when solving problems involving up to 10 objects, and can read and write the numbers involved. They count on and back in steps of different sizes and from different numbers. They measure and order objects using direct comparison, and order events. They are aware of the value of different coins. They use everyday language to compare and to describe positions and properties of regular shapes. They recognise, use and make repeating patterns. They sort and classify objects, demonstrating the criterion they have used.

Level 2

Pupils talk about their work using familiar mathematical language, and represent it using symbols and simple diagrams. They count sets of objects reliably, and use mental recall of number facts to 10 to add or subtract larger numbers. They order numbers up to 100. They choose the appropriate operation when solving addition or subtraction problems. They identify and use halves and quarters in practical situations. They recognise sequences of numbers. They use mental calculation strategies to solve number, money and measure problems. They use everyday non-standard and standard units to measure length and mass. They distinguish between straight and turning movements, recognise half-turns and quarter-turns and right angles in turns. They sort objects and classify them using more than one criterion. When they have gathered information, they record their results in simple lists, tables, diagrams and block graphs.

Level 3

Pupils organise their work, check results, and try different approaches. They talk about and explain their work. They use and interpret mathematical symbols and diagrams. They find particular examples that satisfy a general statement. They use place value in numbers up to 1000 to make approximations. They use decimal notation in recording money, and recognise negative numbers in the context of temperature. They develop further mental strategies for adding and subtracting numbers with at least two digits. They use mental recall of the 2, 3, 4, 5 and 10 multiplication tables in solving whole-number problems involving multiplication and division, including those giving rise to remainders. They use standard units of length, capacity, mass and time. They classify shapes in various ways. They extract and interpret information presented in simple tables and lists, and construct and interpret bar charts and pictograms.

Level 4

Pupils develop their own strategies for solving problems, and present information and results systematically. They search for a solution by trying out ideas of their own. They use their understanding of place value to multiply and divide whole numbers by 10 and 100. They use a variety of mental and written methods for computation, including recall of multiplication facts up to 10×10 . They add and subtract decimals to two places. They check their results are reasonable by considering the context or the size of the numbers. They use simple fractions and percentages to describe approximate parts of a whole. They recognise and describe number patterns and relationships and use simple formulae expressed in words. They use their knowledge of shape to make 3-D mathematical models, draw common 2-D shapes in different orientations on grids, and reflect simple shapes in a mirror line. They choose and use suitable units and instruments, reading, with appropriate accuracy, numbers on a range of measuring instruments. They find perimeters of shapes, areas by counting squares, and volumes by counting cubes. They use and interpret co-ordinates in the first quadrant. They collect discrete data, group data where appropriate, and use the mode and median as characteristics of a set of data. They draw and interpret frequency diagrams and construct and interpret simple line graphs. They understand and use simple vocabulary associated with probability.

Level 5

Pupils identify and obtain information to solve problems, and check whether their results are sensible in the context of the problem. They describe situations mathematically using symbols, words and diagrams and draw their own conclusions, explaining their reasoning. They make general statements of their own, based on available evidence. They use their understanding of place value to multiply and divide whole numbers and decimals. They order, add and subtract negative numbers. They check their solutions by applying inverse operations or estimating using approximations. They calculate fractional or percentage parts of quantities and measurements. They construct and use simple formulae involving one or two operations. They use co-ordinates in all four quadrants. They measure and draw angles to the nearest degree. They recognise, identify and describe all the symmetries of 2-D shapes. They convert one metric unit to another and know the rough metric equivalents of imperial units in daily use. They make sensible estimates of a range of everyday measures. They find areas of rectangles and triangles and volumes of cuboids. They read scales on maps, plans and graphs. They use the mean of discrete data and compare two simple distributions. They interpret graphs, diagrams and pie charts. They use the probability scale from 0 to 1, and appreciate that different outcomes may result from repeating an experiment.

Attainment target

Level 6

Pupils solve complex problems by breaking them down into smaller tasks, and give some mathematical justifications to support their methods, arguments or conclusions. They interpret, discuss and synthesise information presented in a variety of mathematical forms. They use trial-and-improvement methods involving approximating and ordering decimals. They calculate one number as a fraction or percentage of another. They use the equivalences between fractions, decimals and percentages and calculate using ratios in appropriate situations. They find and describe in words the rule for the next term or n th term of a sequence where the rule is linear, and they formulate and solve a variety of simple linear equations. They represent mappings expressed algebraically. They use common 2-D representations of 3-D objects, and the properties of quadrilaterals to classify different types of quadrilateral. They solve problems using angle and symmetry properties of polygons and properties of intersecting and parallel lines. They use formulae for finding circumferences and areas of circles, areas of plane rectilinear figures and volumes of cuboids, and enlarge shapes by a positive whole-number scale factor. They collect and record continuous data, and construct and interpret frequency diagrams, pie charts and scatter diagrams. They use their knowledge that the total probability of all the mutually exclusive outcomes of an experiment is 1, and find and justify probabilities. They identify all the outcomes when dealing with a combination of two experiments.

Level 7

Pupils justify their generalisations, arguments or solutions, consider alternative approaches and appreciate the difference between mathematical explanation and experimental evidence. They examine critically and justify their choice of mathematical presentation. In making estimates, they round to one significant figure and multiply and divide mentally. They understand the effects of multiplying and dividing by numbers between 0 and 1, and calculate proportional changes. They solve numerical problems with numbers of any size, using a calculator efficiently and appropriately. They describe in symbols the next term or n th term of a sequence with a quadratic rule. They use algebraic and graphical methods to solve simultaneous linear equations in two variables and solve simple inequalities. They use Pythagoras' theorem in two dimensions, calculate lengths, areas and volumes in plane shapes and right prisms, and enlarge shapes by a fractional scale factor. They appreciate the imprecision of measurement, and use compound measures such as speed. They specify and test hypotheses, taking account of bias. They analyse data to determine modal class and estimate the mean, median and range of sets of grouped data. They use measures of average and range to compare distributions, and draw a line of best fit on a scatter diagram by inspection. They use relative frequency as an estimate of probability and use this to compare outcomes of experiments.

Level 8

Pupils develop and follow alternative approaches, reflecting on their own lines of enquiry and using a range of mathematical techniques. They examine and discuss generalisations or solutions they have reached. They convey mathematical or statistical meaning through precise and consistent use of symbols. They solve problems involving calculating with the extended number system, including powers, roots and standard form. They manipulate algebraic formulae, equations and expressions. They solve inequalities in two variables. They sketch and interpret graphs of linear, quadratic, cubic and reciprocal functions, and graphs that model real situations. They understand congruence and mathematical similarity, and use sine, cosine and tangent in right-angled triangles. They interpret and construct cumulative frequency tables and diagrams. They compare distributions and make inferences, using estimates of the median and inter-quartile range. They solve problems using the probability of a compound event.

Exceptional Performance

Pupils give reasons for the choices they make when investigating within mathematics. They use mathematical language and symbols effectively in presenting a convincing reasoned argument, including mathematical justification. They express general laws in symbolic form. They solve problems using intersections and gradients of graphs. They use, generate and interpret graphs based on trigonometric functions. They solve problems in two and three dimensions using Pythagoras' theorem and trigonometric ratios. They calculate lengths of circular arcs, areas of sectors, surface areas of cylinders, and volumes of cones and spheres. They interpret and construct histograms. They understand how different sample sizes may affect the reliability of conclusions. They recognise when and how to use conditional probability.

National curriculum outcomes

The following national curriculum outcomes are non-statutory. They have been written to recognise the attainment of pupils working below Level 1. National Curriculum Outcomes 1, 2 and 3 align with the Foundation Phase Outcomes 1, 2 and 3.

Foundation Phase	National Curriculum
Foundation Phase Outcome 1	National Curriculum Outcome 1
Foundation Phase Outcome 2	National Curriculum Outcome 2
Foundation Phase Outcome 3	National Curriculum Outcome 3
Foundation Phase Outcome 4	National Curriculum Level 1
Foundation Phase Outcome 5	National Curriculum Level 2
Foundation Phase Outcome 6	National Curriculum Level 3

The national curriculum outcomes describe the types and range of performance that pupils working at a particular outcome should characteristically demonstrate. In deciding on a pupil's outcome of attainment at the end of a key stage, teachers should judge which description best fits the pupil's performance. Each description should be considered in conjunction with the descriptions for adjacent outcomes.

Outcome 1

Pupils anticipate, follow, respond to and join in with familiar number rhymes, stories, songs, activities and games. They show an awareness of number activities, recite, sign or indicate one or more numbers to five and count or indicate two objects. They are beginning to compare physical properties of objects. They demonstrate interest in position and the relationship between objects. They match objects or pictures by recognising similarities.

Outcome 2

Pupils use mathematics in day-to-day activities and in their play, responding appropriately to key vocabulary and questions. They join in rote counting of numbers from 1 to 10. They recognise and name numbers 1 to 3, and count up to three objects reliably. They record numbers initially by making marks or drawing pictures. They begin to develop an understanding of one-to-one correspondence by matching pairs of different objects or pictures. Pupils understand the concept of 'one more'. In daily activities, they develop an awareness of the purpose of money. They show understanding of words, signs and symbols that describe size and positions. They sort objects using one criterion, and are aware of contrasting qualities.

Outcome 3

Pupils use familiar words in practical situations. They rote count to beyond 10, and onwards from a given small number. They carry out simple addition using numbers 1 to 5 and understand that zero means none. They recognise and try to record numerals from one to nine. They understand the concept of 'one less'. They compare and order two or more objects by direct observation. They show awareness of time in terms of their daily activities. They talk about or indicate, recognise and copy simple repeating patterns and sequences. When sorting, they recognise when an object is different and does not belong to a familiar category.



Key Stage 4

Learning Pathways 14–19

For learners at Key Stage 4, mathematics will be part of each individual's learning pathway. The course of study followed should be designed to encourage both the abilities of young people as learners and their desire to access future learning opportunities. In particular, the course should contribute as widely as possible to the four aspects of learning as identified in the 14–19 Learning Core.

Mathematics at Key Stage 4

At Key Stage 4, learners build on the skills, knowledge and understanding they have already acquired during Key Stage 3, and take on increasing responsibility for planning and executing their work. They bring previous experience and related knowledge to bear when considering mathematical or real-life problems, and extend their understanding of the importance of precision and rigour in mathematics. They communicate their findings with increasing clarity and conciseness when using mathematical language and notation, and demonstrate increasing confidence when using spoken language.

They consolidate their knowledge and understanding of the number system and number relationships, and develop their calculation skills, enabling them to solve problems in a variety of contexts, including work-related and vocational areas, and everyday situations, particularly the management of personal finances. They establish greater proficiency in the use of symbolic manipulation, and rigorous mathematical reasoning in algebraic and geometric contexts. Learners engage in purposeful activities in order to extend their understanding of the use of measures and properties of shape, position, movement and transformation. Through a variety of meaningful contexts, they develop increased confidence in processing and interpreting data, and in understanding and calculating estimates of probabilities.



Skills

Pupils should develop their application and understanding of their mathematical skills using contexts and techniques from across the Range.

1. Solve mathematical problems

Pupils should be given opportunities to:

- select, organise and use the mathematics and resources needed to solve problems of increasing complexity
- identify what further information or data may be required in order to pursue a particular line of enquiry; formulate questions and identify sources of information
- develop and use their own mathematical strategies and ideas creatively and consider those of others
- break complex problems into a series of tasks; select, trial and evaluate a variety of possible approaches, progressing to solving problems set in contexts or areas that are new to them
- use their knowledge of mathematical relationships and structure to derive facts that they have not learned, and to solve mathematical problems
- use a range of mental, written and calculator computational strategies
- use a variety of checking strategies, including mental estimation, approximation and inverse operations
- develop their skills of estimating and measuring; recognise limitations on the accuracy of data and measurement, leading to awareness of the upper and lower bounds of numerical solutions; select an appropriate degree of accuracy.

2. Communicate mathematically

Pupils should be given opportunities to:

- use a wide range of mathematical language, notation, symbols and conventions, to explain their work, communicate findings and express mathematical ideas unambiguously
- generalise, explain patterns and relationships and express functions in words and symbolically
- visualise, describe and represent shapes, movements and transformations, with increasing precision
- read mathematical data in a range of forms
- present work clearly, using diagrams, labelled graphs and symbols
- evaluate different forms of recording and presenting information, taking account of the context and audience
- explain strategies, methods, choices, conclusions and reasoning in a variety of ways.

3. Reason mathematically

Pupils should be given opportunities to:

- extend mental methods of computation to consolidate a range of non-calculator methods
- justify how they arrived at a conclusion to a problem; give solutions in the context of the problem; confirm that results are of the right order of magnitude
- interpret and use algebraic relationships and functions; predict patterns or subsequent terms in sequences
- understand general algebraic statements; make and test generalisations; recognise particular examples of a general statement
- interpret and draw inferences from mathematical information presented in a variety of forms, including graphs, diagrams and statistics; recognise that some conclusions and graphical representations of data can be misleading; examine critically, improve and justify their choice of mathematical presentation
- explain, follow and compare lines of mathematical argument; make conjectures and hypotheses, design methods to test them, and analyse results to see whether they are valid; appreciate the difference between mathematical explanation and experimental evidence; use increasingly more rigorous argument, leading to notions of proof; understand the conditions under which generalisations, inferences and solutions to problems remain valid
- evaluate results by relating them to the initial question or problem; develop an understanding of the reliability of results; recognise that inferences drawn from data analysis may suggest the need for further investigation.



Range

Pupils develop their mathematical skills, knowledge and understanding through learning about and using Number, Measures and money, Algebra, Shape, position and movement, and Handling data. They should use a variety of ICT resources as tools whenever appropriate.

Number

Pupils should be given opportunities to:

1. Understand number and number notation

- extend their knowledge of the number system, including the distinction between rational and irrational numbers
- use place value in whole numbers and decimals in computation and metric measurement
- use negative numbers in a variety of contexts
- use index notation and standard form

2. Calculate in a variety of ways

- use a calculator efficiently to plan a complex calculation
- calculate with whole numbers, negative numbers, decimals, fractions, percentages and ratios, understanding the effects of the operations
- use direct and inverse proportion
- simplify numerical expressions involving surds and use indices with negative and fractional values

3. Investigate patterns and relationships

- explore features of numbers, including primes, powers and roots

- explore patterns and sequences of numbers
- explore inverse number relationships, and use these in computation strategies.

Measures and money

Pupils should be given opportunities to:

1. Understand and use measures

- extend their understanding of the nature of measurement, including the difference between discrete and continuous measures
- make sensible estimates of length, mass, capacity and time in everyday situations, extending to less familiar contexts
- develop their understanding of the relationships between units, converting from one metric unit to another
- know imperial measures in common use and their approximate metric equivalents
- use compound measures
- use and interpret scale on graphs, maps and drawings
- read and interpret scales on measuring instruments and understand the degree of accuracy that is possible, or appropriate, for a given purpose
- calculate lengths, angles, perimeters, areas and volumes associated with common shapes, progressing to more complex plane shapes and solids, including sectors, cylinders, cones and spheres; use the relationships between similar figures and solids
- distinguish between formulae by considering dimensions

2. Understand and use money

- understand and use the conventional way of recording money
- calculate with money and solve problems related to budgeting, saving and spending, including currency exchange rates, profit and loss, discount, hire purchase, best buys, household bills and compound interest
- interpret a calculator display in relation to money.



Algebra

Pupils should be given opportunities to:

1. Understand and use algebraic relationships, functions and graphs

- extend their knowledge of number operations and relationships to develop the ideas of algebra; appreciate the use of letters to represent variables or unknowns
- experience ways in which algebra can be used to model real-life situations and solve problems
- generate and generalise number sequences
- construct and interpret tables and graphs of linear, quadratic, simple cubic and reciprocal functions; explore the properties of these functions and their graphs
- construct, interpret, simplify and evaluate formulae and expressions given in words or symbols
- form, manipulate and solve linear equations and inequalities, linear simultaneous equations, quadratic and simple cubic equations, including trial-and-improvement methods where appropriate; solve equations and inequalities by algebraic and graphical methods, selecting the most appropriate method for the problem concerned

- interpret and apply the transformation of functions in the context of their graphical representation, including $y = f(x + a)$, $y = f(kx)$ and $y = f(x) + a$, applied to $y = f(x)$
- construct tangents to curves and interpret their gradients; interpret the meaning of the area under a graph; apply these to the solution of numerical and statistical problems, and those involving distance–time and velocity–time graphs.

Shape, position and movement

Pupils should be given opportunities to:

1. Understand and use the properties of shapes

- explore properties of shapes through drawing and practical work; construct 2-D and 3-D shapes from given information
- understand congruence of 2-D shapes
- explain and use angle properties of 2-D shapes; use angle and tangent properties of circles
- explore the properties of polygons and polyhedra; use these to classify shapes
- use Pythagoras' theorem to solve problems
- understand the trigonometrical relationships in right-angled triangles and use these to solve problems in two and three dimensions
- extend their understanding of trigonometry to angles of any size, the graphs and behaviour of trigonometric functions, and the application of these to the solution of problems in two dimensions, using the sine and cosine rules



2. Understand and use the properties of position and movement

- use line and rotational symmetries to solve problems in two and three dimensions
- use Cartesian co-ordinates to specify location
- develop their understanding of tessellations and geometric patterns, the transformations of translation, reflection, rotation and enlargement, and their combination in two dimensions
- determine the locus of an object moving according to a given rule.

Handling data

Pupils should be given opportunities to:

1. Collect, represent, analyse and interpret data

- specify the problem clearly, plan and collect data appropriately in order to follow lines of enquiry or to test hypotheses
- develop an understanding of bias and reliability
- construct appropriate diagrams and graphs to represent discrete and continuous data, including bar charts, line graphs, pie charts, frequency polygons, scatter diagrams, lines of best fit, cumulative frequency diagrams and histograms
- calculate or estimate values of the mode, median, mean, range and inter-quartile range of discrete, grouped and continuous data
- interpret information given in a wide range of graphs, diagrams and statistics, particularly real-life data
- compare sets of data and their distributions, using appropriate methods, including those that involve describing central tendency, dispersion and correlation

2. Understand and use probability

- understand and use the vocabulary of probability and the probability scale from 0 to 1 through experience, experiment and theory
- recognise situations where probabilities can be based on equally likely outcomes and others where estimates must be based on experimental evidence; calculate and make these estimates as appropriate, using relative frequency over a number of trials as an estimate of probability
- identify all the outcomes of a combination of two experiments or a compound event, progressing to the calculation of probabilities, including the use of tree diagrams
- recognise the conditions for the addition of probabilities for mutually exclusive events, and the multiplication of probabilities for two independent events, and make the appropriate calculations when these conditions apply
- understand when and how to estimate conditional probabilities.